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MATHEMATICAL MODEL OF OSCILLATIONS OF A THREE-LAYERED CHANNEL WALL POSSESSING A COMPRESSIBLE CORE AND INTERACTING WITH A PULSATING VISCOUS LIQUID LAYER

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Abstract

The paper deals with the formulation of a mathematical model to study a dynamics interaction of a three-layered channel wall with a pulsating viscous fluid layer in a channel. The narrow channel formed by two parallel walls was considered. The lower channel wall was a three-layered plate with a compressible core, and the upper one was absolutely rigid. The face sheets of the three-layered plate satisfied Kirchhoff's hypotheses. The plate core was considered rigid taking into account its compression in the transverse direction. Plate deformations were assumed to be small. The continuity conditions of displacements are satisfied at the layers' boundaries of the three-layered plate. The oscillations of the three-layered channel wall occurred under the action of a given law of pressure pulsation at the channel edges. The dynamics of the viscous incompressible fluid layer within the framework of a creeping motion was considered. The formulated mathematical model consisted of the dynamics equations of the three-layered plate with compressible core, Navier — Stokes equations, and the continuity equation. The boundary conditions of the model were the conditions at the plate edges, the no-slip conditions at the channel walls and the conditions for pressure at the channel edges. The steady-state harmonic oscillations were investigated and longitudinal displacements and deflections of the plate face sheets were determined.

Keywords

Modeling, oscillations, hydroelasticity, three-layered plate, compressible core, viscous liquid

Frequency-dependent distribution functions of amplitudes of plate layers displacements were introduced. These functions allow us to investigate the dynamic response of the channel wall and the fluid pressure change in the channel. The elaborated model can be used for the evolution of non-destructive testing of elastic three-layered elements contacting with a viscous fluid layer and being part of the lubrication, damping or cooling systems of modern instruments and units

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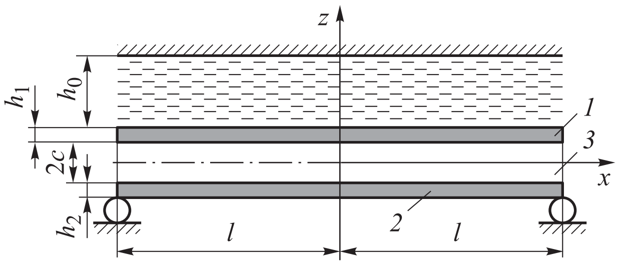
Introduction. Modeling the interaction of elastic elements of structures with a fluid is of great importance for the development of instrument making. For example, in one of the first papers [1], in which free vibrations of a circular plate in contact with water were considered, its author H. Lamb noted that this study is extremely important for transmitting signals underwater. Using Rayleigh strain-energy method, he obtained expressions for the plate's oscillation frequencies. Further development of this issue was carried out in [2] based on of the formulation and solution of the hydroelasticity problem in joint consideration of the equations of motion of an elastic element and fluid. In these works, autors relied on the contact of one plate surface with an unlimited volume of an ideal incompressible fluid, and it was shown that oscillation is damped due to the conversion of energy to wave formation in a liquid, and the effect of increasing inertia, estimated by the added mass, is observed, resulting in a decrease in vibration frequencies. Further studies on this issue are aimed at studying the influence of related factors. For example, the study of the attached masses of a fluid, which take into account its inertial properties, with oscillations of plates of different shapes and with various methods of their fixing was performed in [3]. In [4], free vibrations of a circular plate on the free surface of an ideal non-compressible fluid, the volume of which is limited by a rigid cylindrical wall and bottom, were investigated. A similar problem with the immersion of a plate under the free surface of an ideal fluid was considered in [5]. The problem of chaotic oscillations of a plate that is in contact on both sides with the flow of an ideal fluid is considered in [6]. Mathematical models for the study of the dynamics of elastic elements of pressure sensors and vibration devices based on the formulation of the problem of hydroelasticity of a plate interacting with an ideal fluid were considered in [7, 8]. In [9], a model was proposed for determining the width of the contact zone between solid surfaces and a thinwalled cooling channel, having a flat-oval section, when it is hydroelastic deformed under the influence of internal static pressure.

At the same time, studies of vibrations of elastic elements of structures interacting with a viscous fluid are extremely important, since viscosity determines the damping properties in an oscillatory system. For example, in [10, 11], the interaction of a viscous fluid with the structural elements of gyroscopic devices was considered and the effect of this interaction on the vibration resistance and accuracy of devices was shown. In [12], the study [1] was generalized for the case of taking into account the viscosity of the fluid. Hydroelastic vibrations of the elastic elements of the vibratory machine interacting with a layer of a viscous fluid are considered in [13]. The problem of oscillations of a cantilever-fixed beam immersed in an unlimited volume of a viscous fluid was solved in [14]. A similar problem for a piezoelectric beam in a viscous fluid flow was studied in [15]. The transverse oscillations of two coaxial disks interacting with a layer of a viscous incompressible fluid between them are studied in [16].

In connection with the development of aerospace technology, the three-layered structural elements in the form of beams and plates are becoming more widely used in modern aggregate instrument making. These elements have the necessary rigidity and low weight, and protect against aggressive effects (temperature, radiation, etc.). Approaches to the study of their statics and dynamics are well developed and presented, for example, in the review part of the monograph [17]. However, studies on the simulation of their oscillations in the interaction with the liquid are not enough. The oscillations of composite plates interacting with an ideal liquid were investigated in [18]. In [19–22], vibrations of three-layered beams and plates prossessing an incompressible core and interacting with a viscous fluid were studied. In this paper, a mathematical model is proposed for studying the longitudinal and bending vibrations of a three-layered channel wall having a compressible core and interacting with a pulsating layer of viscous fluid, taking into account normal and tangential stresses on its side.

Formulation of the problem. Consider a narrow channel formed by two parallel walls, the bottom wall of which is a three-layered plate with compressible filler (Figure). The plate is simply supported on the ends, the thickness of its outer bearing layers are h_1 and h_2, the thickness of the core is $2c$. We (or let us) associate the Cartesian coordinate system xyz with the middle plane of the core in the undisturbed state. The upper wall of the channel is rigid. We assume the size of the channel in terms of $2l \times b$ and assume $b \gg 2l$, that is, we consider the plane problem. The distance between the channel walls in the undisturbed state is

h_0 and $2l \gg h_0$. The channel is filled with a viscous incompressible fluid, the pressure in which pulses due to the harmonic law of pressure pulsation given at the ends $p^* = p_0 + p_m \sin(\omega t)$. Here p_0 is constantly pressure level, p_m is amplitude of pressure pulsation, ω is frequency, t is time. The elastic movement of the plate is much less than h_0. We investigate the steady-state oscillations since the viscosity of the fluid cause's considerable frictional forces in the channel, which leads to a rapid decay of transients.



A narrow channel, the bottom wall of which is a three-layered plate:
1, *2* are upper and lower bearing layers; *3* is core

The plate consists of upper and lower bearing layers, which perceive the main loads, and core, ensuring their joint work as a single package. We assume [17] that the bearing layers are isotropic, incompressible in the transverse direction, and satisfy the Kirchhoff conjectures. The core is considered hard given its compression, the exact relations of the theory of elasticity hold for it, and the dependence of the displacements of its points on the transverse coordinate z appears linear. The deformations are assumed to be small, and at the boundaries of the layers of the plate, the conditions for the continuity of their displacements are satisfied. For the above assumptions, the stress-deformable state of the plate is fully described by the longitudinal displacements and deflections of the middle planes of its bearing layers, and the equations of its dynamics are as follows [17]:

$$\begin{aligned} &F_1 + a_1 u_1 - a_1 u_2 - a_4 \frac{\partial^2 u_1}{\partial x^2} - a_5 \frac{\partial^2 u_2}{\partial x^2} + a_2 \frac{\partial w_1}{\partial x} + \\ &+ a_3 \frac{\partial w_2}{\partial x} - 2 a_6 \frac{\partial^3 w_1}{\partial x^3} + a_7 \frac{\partial^3 w_2}{\partial x^3} = q_{zx}; \\ &F_2 - a_1 u_1 + a_1 u_2 - a_5 \frac{\partial^2 u_1}{\partial x^2} - a_9 \frac{\partial^2 u_2}{\partial x^2} - a_3 \frac{\partial w_1}{\partial x} - \\ &- a_2 \frac{\partial w_2}{\partial x} - a_6 \frac{\partial^3 w_1}{\partial x^3} + 2 a_7 \frac{\partial^3 w_2}{\partial x^3} = 0; \end{aligned} \qquad (1)$$

$$F_3 - a_{17}\frac{\partial u_1}{\partial x} + a_{10}\frac{\partial u_2}{\partial x} + 2a_6\frac{\partial^3 u_1}{\partial x^3} + a_6\frac{\partial^3 u_2}{\partial x^3} + a_{11}\frac{\partial^2 w_1}{\partial x^2} - a_{12}\frac{\partial^2 w_2}{\partial x^2} +$$
$$+ a_{15}\frac{\partial^4 w_1}{\partial x^4} - 2a_{16}\frac{\partial^4 w_2}{\partial x^4} + a_8 w_1 - a_8 w_2 = q_{zz} + \frac{1}{2}h_1\frac{\partial q_{zx}}{\partial x};$$
$$F_4 - a_{18}\frac{\partial u_1}{\partial x} + a_{19}\frac{\partial u_2}{\partial x} - a_7\frac{\partial^3 u_1}{\partial x^3} - 2a_7\frac{\partial^3 u_2}{\partial x^3} - a_{12}\frac{\partial^2 w_1}{\partial x^2} + a_{14}\frac{\partial^2 w_2}{\partial x^2} -$$
$$- a_{16}\frac{\partial^4 w_1}{\partial x^4} + a_{13}\frac{\partial^4 w_2}{\partial x^4} - a_8 w_1 + a_8 w_2 = 0.$$

Here u_1 and u_2 are the elastic longitudinal displacements of the upper and lower bearing layers of the plate, w_1 and w_2 are the deflections of the upper and lower bearing layers of the plate, q_{zz}, q_{zx} are the normal and shear stresses acting on the upper supporting layer of the plate from the liquid side, these stresses are written as

$$q_{zx} = -\rho\nu\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right) \text{ at } z = c + h_1 + w_1;$$
$$q_{zz} = -p + 2\rho\nu\frac{\partial u_z}{\partial z} \text{ at } z = c + h_1 + w_1, \qquad (2)$$

where ρ, ν are the density and kinematic viscosity of a fluid, u_x, u_z are the projections of the velocity vector of the fluid on the coordinate axes, p is the fluid pressure, ρ_k is the material density of the k-th plate layer, G_k, K_k are the shear and volume strain moduli of the k-th plate layer, $K_k^+ = K_k + \frac{4}{3}G_k$, $K_k^- = K_k - \frac{4}{3}G_k$.

In addition, the following notation has been introduced:

$$a_1 = \frac{2G_3}{c};\; a_2 = 2G_3\left(1 + \frac{h_1}{2c}\right) - \frac{K_3^-}{2};\; a_3 = 2G_3\left(1 + \frac{h_2}{2c}\right) + \frac{K_3^-}{2};$$

$$a_4 = K_1^+ h_1 + \frac{2K_3^+ c}{3};\; a_5 = \frac{K_3^+ c}{3};\; a_6 = \frac{K_3^+ c h_1}{6};\; a_7 = \frac{K_3^+ c h_2}{6};$$

$$a_8 = \frac{K_3^+}{2c};\; a_9 = K_2^+ h_2 + \frac{2K_3^+ c}{3};\; a_{10} = \frac{G_3}{2}\left(1 + \frac{h_1}{2c}\right) + \frac{K_3^-}{2};$$

$$a_{11} = \frac{K_3^- h_1}{2} - \frac{G_3 c}{2}\left(1 + \frac{h_1}{2c}\right)^2 - \frac{G_3 c}{6};$$

$$a_{12}=\frac{K_3^-\left(h_1+h_2\right)}{4}+\frac{G_3\,c}{2}\left(1+\frac{h_1}{2c}\right)\left(1+\frac{h_2}{2c}\right)-\frac{G_3\,c}{6};$$

$$a_{13}=\frac{K_2^+\,h_2^3}{12}+\frac{K_3^+\,c\,h_2^2}{6};\;\; a_{14}=\frac{K_3^-\,h_2}{2}-\frac{G_3\,c}{2}\left(1+\frac{h_2}{2c}\right)^2-\frac{G_3\,c}{6};$$

$$a_{15}=\frac{K_1^+\,h_1^3}{12}+\frac{K_3^+\,c\,h_1^2}{6};\;\; a_{16}=\frac{K_3^+\,c\,h_2\,h_1}{12};\;\; a_{17}=\frac{G_3}{2}\left(1+\frac{h_1}{2c}\right)-\frac{K_3^-}{2};$$

$$a_{18}=\frac{G_3}{2}\left(1+\frac{h_2}{2c}\right)+\frac{K_3^-}{2};\;\; a_{19}=\frac{G_3}{2}\left(1+\frac{h_2}{2c}\right)-\frac{K_3^-}{2};$$

$$F_1=\frac{\partial^2}{\partial t^2}\left(m_1\,u_1+m_8u_2+2\,m_5\,\frac{\partial w_1}{\partial x}-m_7\,\frac{\partial w_2}{\partial x}\right);$$

$$F_2=\frac{\partial^2}{\partial t^2}\left(m_8\,u_1+m_2u_2+m_5\,\frac{\partial w_1}{\partial x}-2\,m_7\,\frac{\partial w_2}{\partial x}\right);$$

$$F_3=\frac{\partial^2}{\partial t^2}\left(-2\,m_5\,\frac{\partial u_1}{\partial x}-m_5\,\frac{\partial u_2}{\partial x}+m_1\,w_1+m_8\,w_2-m_3\,\frac{\partial^2 w_1}{\partial x^2}+m_6\,\frac{\partial^2 w_2}{\partial x^2}\right);$$

$$F_4=\frac{\partial^2}{\partial t^2}\left(m_7\,\frac{\partial u_1}{\partial x}+2m_7\,\frac{\partial u_2}{\partial x}+m_8\,w_1+m_2\,w_2+m_6\,\frac{\partial^2 w_1}{\partial x^2}-m_4\,\frac{\partial^2 w_2}{\partial x^2}\right);$$

$$m_1=\rho_1\,h_1+\frac{2}{3}\rho_3\,c;\;\; m_2=\rho_2\,h_2+\frac{2}{3}\rho_3\,c;\;\; m_3=\frac{\rho_1\,h_1^3}{12}+\frac{\rho_3\,c\,h_1^2}{6};$$

$$m_4=\frac{\rho_2 h_2^3}{12}+\frac{\rho_3 c h_2^2}{6};\;\; m_5=\frac{\rho_3 c\,h_1}{6};$$

$$m_6=\frac{\rho_3\,c\,h_1\,h_2}{12};\;\; m_7=\frac{\rho_3\,c\,h_2}{6};\;\; m_8=\frac{\rho_3\,c}{3}.$$

The boundary conditions of equations (1) are

$$w_k=\frac{\partial u_k}{\partial x}=\frac{\partial^2 w_k}{\partial x^2}=0\;\text{ at }\; x=\pm\,l,\;\; k=1,2. \qquad (3)$$

In narrow channels and slots for modeling the dynamics of a viscous fluid, creeping motion [23] can be considered and the equations of motion of the fluid can be written as

$$\frac{1}{\rho}\frac{\partial p}{\partial x}=\nu\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial z^2}\right);\;\; \frac{1}{\rho}\frac{\partial p}{\partial z}=\nu\left(\frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial z^2}\right);\;\; \frac{\partial u_x}{\partial x}+\frac{\partial u_z}{\partial z}=0. \qquad (4)$$

Equations (4) are supplemented by the boundary conditions of fluid adhesion on an absolutely solid wall and the upper bearing layer of the plate

$$u_x = 0,\ u_z = 0 \text{ at } z = c + h_1 + h_0,$$
$$u_x = \frac{\partial u_1}{\partial t},\ u_z = \frac{\partial w_1}{\partial t} \text{ at } z = c + h_1 + w_1, \tag{5}$$

and boundary conditions of coincidence of pressure in a fluid with a given pressure in the end sections of the channel

$$p = p_0 + p^* \text{ at } x = \pm l. \tag{6}$$

Determination of the dynamic response of the three-layer channel wall. We introduce the following dimensionless variables:

$$\tau = \omega t,\ \xi = \frac{x}{l},\ \zeta = \frac{z - c - h_1}{h_0},\ u_x = w_{1m}\,\omega \frac{l}{h_0} U_\xi,$$
$$u_z = w_{1m}\,\omega U_\zeta,\ p = p^* + \frac{\rho\nu w_{1m}\omega}{h_0}\frac{l^2}{h_0^2} P, \tag{7}$$

and given the equations (5) and boundary conditions (6), (7), we obtain

$$\frac{\partial P}{\partial \xi} = \left(\frac{h_0}{l}\right)^2 \frac{\partial^2 U_\xi}{\partial \xi^2} + \frac{\partial^2 U_\xi}{\partial \zeta^2};$$
$$\frac{\partial P}{\partial \zeta} = \left(\frac{h_0}{l}\right)^2 \left[\left(\frac{h_0}{l}\right)^2 \frac{\partial^2 U_\zeta}{\partial \xi^2} + \frac{\partial^2 U_\zeta}{\partial \zeta^2}\right]; \tag{8}$$
$$\frac{\partial U_\xi}{\partial \xi} + \frac{\partial U_\zeta}{\partial \zeta} = 0;$$

$$U_\xi = 0;\ U_\zeta = 0 \text{ at } \zeta = 1;\ U_\xi = \frac{h_0}{\ell}\frac{u_{1m}}{w_{1m}}\frac{\partial U_1}{\partial \tau};$$
$$U_\zeta = \frac{\partial W_1}{\partial \tau} \text{ at } \zeta = \frac{w_{1m}}{h_0}\frac{\partial W_1}{\partial \tau}; \tag{9}$$
$$P = 0 \text{ at } \xi = \pm 1.$$

Here, it was assumed, where $u_1 = u_{1m} U_1(\xi, \tau)$, $w_1 = w_{1m} W_1(\xi, \tau)$, where u_{1m}, w_{1m} are the amplitudes of the longitudinal displacement and deflection of the upper bearing layer.

In the considered formulation $u_{1m}/w_{1m} \approx 1$, and parameters $h_0/l \ll 1$, $w_{m1}/h_0 \ll 1$, i.e., with the given parameters in (8) and (9), can be omitted, as a result we have

$$\frac{\partial P}{\partial \xi}=\frac{\partial^2 U_\xi}{\partial \zeta^2};\quad \frac{\partial P}{\partial \zeta}=0;\quad \frac{\partial U_\xi}{\partial \xi}+\frac{\partial U_\zeta}{\partial \zeta}=0; \tag{10}$$

$$U_\xi=0,\ U_\zeta=0 \text{ at } \zeta=1;\ U_\xi=0,\ U_\zeta=\frac{\partial W_1}{\partial \tau} \text{ at } \zeta=0; \tag{11}$$

$$P=0 \text{ at } \xi=\pm 1. \tag{12}$$

Taking into account the comments made earlier, the stresses q_{zz} and q_{zx} in the variables (8) are written as

$$\begin{aligned} q_{zx}&=-(\rho\nu\, w_{1m}\,\omega l/h_0^2)\,\partial U_\xi\ /\partial\zeta\big|_{\zeta=0};\\ q_{zz}&=-p^*-(\rho\nu\, w_{1m}\,\omega l^2/h_0^3)\,P\big|_{\zeta=0}. \end{aligned} \tag{13}$$

Solving problem (11)–(13), we find that

$$\begin{aligned} P&=12\int_\xi^1\int_0^\xi \frac{\partial W_1}{\partial \tau}d\xi d\xi+6(\xi-1)\int_{-1}^1\int_0^\xi \frac{\partial W_1}{\partial \tau}d\xi\, d\xi;\\ \left.\frac{\partial U_\xi}{\partial \zeta}\right|_{\zeta=0}&=6\int_0^\xi \frac{\partial W_1}{\partial \tau}d\xi-3\int_{-1}^1\int_0^\xi \frac{\partial W_1}{\partial \tau}d\xi d\xi;\\ \frac{h_1}{l}\frac{\partial}{\partial \xi}\left(\frac{\partial U_\xi}{\partial \zeta}\right)\bigg|_{\zeta=0}&=\frac{h_1}{l}6\frac{\partial W_1}{\partial \tau}. \end{aligned} \tag{14}$$

The solution of equations (1) is represented as

$$\begin{aligned} u_k&=\sum_{n=0}^{\infty}T_k^n(\omega t)\cos\big((2n+1)\pi\,\xi/2\big);\\ w_k&=\sum_{n=0}^{\infty}\cos\big((2n+1)\pi\,\xi/2\big),\quad k=1,2. \end{aligned} \tag{15}$$

Substituting (15) into (14) and decomposing the pressure p^* in a series of $\cos\big((2n+1)\pi\xi/2\big)$, we obtain

$$q_{zz}=\sum_{n=0}^{\infty}\frac{\rho\nu l^2}{h_0^3}\left(\frac{4(-1)^{n+1}}{(2n+1)\pi}p^*\frac{h_0^3}{\rho\nu l^2}-12\left[\frac{2}{(2n+1)\pi}\right]^2\frac{dR_1^n}{dt}\right)\cos\frac{2n+1}{2}\pi\frac{x}{l};$$

$$q_{zx}=-\sum_{n=0}^{\infty}\frac{\rho\nu l}{h_0^2}6\left[\frac{2}{(2n+1)\pi}\right]\frac{dR_1^n}{dt}\sin\frac{2n+1}{2}\pi\frac{x}{l}; \tag{16}$$

$$\frac{1}{2}h_1\frac{\partial q_{zx}}{\partial x}=\frac{1}{2}\frac{h_0}{l}\frac{h_1}{l}\sum_{n=0}^{\infty}\frac{\rho\nu l^2}{h_0^3}6\frac{dR_1^n}{dt}\cos\frac{2n+1}{2}\pi\frac{x}{l}.$$

From expressions (16) it follows that $\frac{h_1}{2}\frac{\partial q_{zx}}{\partial x}\Big/ q_{zz} \ll 1$, i.e., the term $\frac{h_1}{2}\frac{\partial q_{zx}}{\partial x}$ in (1) can be neglected in compression with q_{zz}. Then, substituting (15), (16) in (1), and equating the terms in the resulting system with the same trigonometric functions, we proceed to the system of ordinary differential equations in time, which includes two homogeneous algebraic equations. Using them, we will find a connection T_2^n, R_2^n through T_1^n, R_1^n:

$$T_2^n = \left(T_1^n\left(b_{24}b_{41} - b_{44}b_{21}\right) + R_1^n\left(b_{24}b_{43} - b_{44}b_{23}\right)\right)\Big/\left(b_{22}b_{44} - b_{24}b_{42}\right);$$
$$R_2^n = \left(T_1^n\left(b_{42}b_{21} - b_{22}b_{41}\right) + R_1^n\left(b_{42}b_{23} - b_{22}b_{43}\right)\right)\Big/\left(b_{22}b_{44} - b_{24}b_{42}\right). \qquad (17)$$

Further, given that for harmonic steady-state oscillations $d^2R_1^n\big/dt^2 = -\,\omega^2 R_1^n$, we finally get

$$b_{11}^* T_1^n + b_{13}^* R_1^n + 2K_n^1 \frac{dR_1^n}{dt} = 0;$$
$$b_{31}^* T_1^n + b_{33}^* R_1^n + 2K_n \frac{dR_1^n}{dt} = \frac{4(-1)^{n+1}}{(2n+1)\pi} p^*. \qquad (18)$$

The following notation is introduced here:

$$\Delta = b_{22}b_{44} - b_{24}b_{42};$$
$$b_{11}^* = b_{11} + b_{12}\left(b_{24}b_{41} - b_{44}b_{21}\right)/\Delta + b_{14}\left(b_{42}b_{21} - b_{22}b_{41}\right)/\Delta;$$
$$b_{13}^* = b_{12}\left(b_{24}b_{43} - b_{44}b_{23}\right)/\Delta + b_{13} + b_{14}\left(b_{42}b_{23} - b_{22}b_{43}\right)/\Delta;$$
$$b_{31}^* = b_{31} + b_{32}\left(b_{24}b_{41} - b_{44}b_{21}\right)/\Delta + b_{34}\left(b_{42}b_{21} - b_{22}b_{41}\right)/\Delta;$$
$$b_{33}^* = b_{32}\left(b_{24}b_{43} - b_{44}b_{23}\right)/\Delta + b_{33} + b_{34}\left(b_{42}b_{23} - b_{22}b_{43}\right)/\Delta;$$

$$2K_n = 12\,\frac{\rho\nu l^2}{h_0^3}\left[\frac{2}{(2n+1)\pi}\right]^2;\; 2K_n^1 = 6\,\frac{\rho\nu l}{h_0^2}\left[\frac{2}{(2n+1)\pi}\right];$$

$$b_{11} = a_1 + a_4\left(\frac{2n+1}{2l}\pi\right)^2 - m_1\omega^2;\; b_{12} = -a_1 + a_5\left(\frac{2n+1}{2l}\pi\right)^2 - m_8\omega^2;$$

$$b_{13} = \frac{2n+1}{2l}\pi\left[-a_2 - 2a_6\left(\frac{2n+1}{2l}\pi\right)^2 + 2m_5\omega^2\left(\frac{2n+1}{2l}\pi\right)\right];$$

$$b_{14}=\frac{2n+1}{2l}\pi\left[-a_3+a_7\left(\frac{2n+1}{2l}\pi\right)^2-m_7\,\omega^2\right];$$

$$b_{21}=-a_1+a_5\left(\frac{2n+1}{2l}\pi\right)^2-m_8\omega^2;$$

$$b_{22}=a_1+a_9\left(\frac{2n+1}{2l}\pi\right)^2-m_2\omega^2;$$

$$b_{23}=\frac{2n+1}{2l}\pi\left[a_3-a_6\left(\frac{2n+1}{2l}\pi\right)^2+m_5\omega^2\right];$$

$$b_{24}=\frac{2n+1}{2l}\pi\left[a_2+2a_7\left(\frac{2n+1}{2l}\pi\right)^2-2m_7\omega^2\right];$$

$$b_{31}=\frac{2n+1}{2l}\pi\left[a_{17}-2\,a_6\left(\frac{2n+1}{2l}\pi\right)^2+2m_5\,\omega^2\right];$$

$$b_{32}=\frac{2n+1}{2l}\pi\left[a_{10}-a_6\left(\frac{2n+1}{2l}\pi\right)^2+m_5\,\omega^2\right];$$

$$b_{33}=a_8-a_{11}\left(\frac{2n+1}{2l}\pi\right)^2+a_{15}\left(\frac{2n+1}{2l}\pi\right)^4-\left[m_1+m_3\left(\frac{2n+1}{2l}\pi\right)^2\right]\omega^2;$$

$$b_{34}=-a_8+a_{12}\left(\frac{2n+1}{2l}\pi\right)^2-a_{16}\left(\frac{2n+1}{2l}\pi\right)^2-\left[m_8-m_6\left(\frac{2n+1}{2l}\pi\right)^2\right]\omega^2;$$

$$b_{41}=\frac{2n+1}{2l}\pi\left[-a_{18}+a_7\left(\frac{2n+1}{2l}\right)^2-m_7\,\omega^2\right];$$

$$b_{42}=\frac{2n+1}{2l}\pi\left[a_{19}+2\,a_7\left(\frac{2n+1}{2l}\pi\right)^2-2m_7\omega^2\right];$$

$$b_{43}=-a_8+a_{12}\left(\frac{2n+1}{2l}\pi\right)^2-a_{16}\left(\frac{2n+1}{2l}\pi\right)^4-\left[m_8-m_3\left(\frac{2n+1}{2l}\pi\right)^2\right]\omega^2;$$

$$b_{44}=a_8-a_{14}\left(\frac{2n+1}{2l}\pi\right)^2+a_{13}\left(\frac{2n+1}{2l}\pi\right)^4-\left[m_2+m_4\left(\frac{2n+1}{2l}\pi\right)^2\right]\omega^2.$$

From (18) we have

$$\frac{dR_1^n}{dt}\left[\frac{2K_n}{b_{31}^*}-\frac{2K_n^1}{b_{11}^*}\right]+R_1^n\left[\frac{b_{33}^*}{b_{31}^*}-\frac{b_{13}^*}{b_{11}^*}\right]=\frac{4(-1)^{n+1}}{(2n+1)\,\pi}\frac{1}{b_{31}^*}\,p^*;$$
$$T_1^n=-\frac{b_{13}^*}{b_{11}^*}R_1^n-\frac{2K_n^1}{b_{11}^*}\frac{dR_1^n}{dt}. \qquad (19)$$

Given the linearity (19) and representing $R_1^n = R_1^{n0}+\bar{R}_1^n$, $T_1^n = T_1^{n0}+\bar{T}_1^n$, where the superscript 0 corresponds to a static solution, we have

$$R_1^{n0}=p_0\,\frac{4(-1)^{n+1}}{(2n+1)\pi}\left.\frac{1}{d_1}\right|_{\omega=0};$$
$$T_1^{n0}=-R_1^{n0}\left.\frac{b_{13}^*}{b_{11}^*}\right|_{\omega=0}=p_0\,\frac{4(-1)^n}{(2n+1)\,\pi}\left.\left(\frac{b_{13}^*}{b_{11}^*}\frac{1}{d_1}\right)\right|_{\omega=0} \qquad (20)$$

and for harmonic oscillations we find that

$$\bar{R}_1^n=\frac{4(-1)^{n+1}}{(2n+1)\,\pi}\frac{1}{\sqrt{d_1^2+d_2^2\omega^2}}\,e^{i\psi}\,p_m^*\,e^{i\omega t};$$
$$\bar{T}_1^n=\frac{4(-1)^n}{(2n+1)\,\pi}\frac{\sqrt{b_{13}^{*2}+(2K_n^1\omega)^2}}{\sqrt{d_1^2b_{11}^{*2}+d_2^2\omega^2b_{11}^{*2}}}\,e^{i\theta}\,e^{i\psi}\,p_m^*\,e^{i\omega t}, \qquad (21)$$

where $d_1 = b_{33}^*-b_{31}^*b_{13}^*/b_{11}^*$; $d_2 = 2K_n - b_{31}^*2K_n^1/b_{11}^*$; $\operatorname{tg}\psi=-d_2\omega/d_1$; $\operatorname{tg}\theta = 2K_n^1\omega/b_{13}^*$.

Taking into account (20) and (21) in (15), we obtain expressions for the deflection and longitudinal displacement of the first bearing layer of the plate during its oscillations:

$$w_1=p_0\sum_{n=0}^{\infty}\frac{4(-1)^{n+1}}{(2n+1)\,\pi}\left.\frac{1}{d_1}\right|_{\omega=0}\cos\frac{2n+1}{2l}\pi x+$$
$$+\,p_m^*\Pi_{w1}(\omega,x)\sin(\omega t+\varphi_{w1});$$
$$u_1=p_0\sum_{n=0}^{\infty}\frac{4(-1)^n}{(2n+1)\,\pi}\left.\left(\frac{b_{13}^*}{b_{11}^*}\frac{1}{d_1}\right)\right|_{\omega=0}\sin\frac{2n+1}{2l}\pi x+$$
$$+\,p_m^*\Pi_{u1}(\omega,x)\sin(\omega t+\varphi_{u1}). \qquad (22)$$

Here

$$\Pi_{w1}(\omega,x)=\sqrt{E_p^2+F_p^2};\quad \Pi_{u1}(\omega,x)=\sqrt{A_p^2+B_p^2};$$

$$E_p = \sum_{n=0}^{\infty} \frac{4(-1)^{n+1}}{(2n+1)\,\pi} \frac{d_1}{d_1^2 + d_2^2\omega^2} \cos\frac{2n+1}{2l}\pi x;$$

$$F_p = \sum_{m=0}^{\infty} \frac{4(-1)^{n+1}}{(2n+1)\,\pi} \frac{\omega d_2}{d_1^2 + d_2^2\omega^2} \cos\frac{2n+1}{2l}\pi x;$$

$$A_p = \sum_{n=0}^{\infty} \frac{4(-1)^{n}}{(2n+1)\,\pi} \left[\frac{d_1}{d_1^2 + d_2^2\omega^2} \frac{b_{13}^*}{b_{11}^*} + \frac{2K_n^1\omega}{b_{11}^*} \frac{d_2\,\omega}{d_1^2 + d_2^2\omega^2} \right] \sin\frac{2n+1}{2l}\pi x;$$

$$B_p = \sum_{n=0}^{\infty} \frac{4(-1)^{n}}{(2n+1)\,\pi} \left[\frac{d_1}{d_1^2 + d_2^2\omega^2} \frac{2K_n^1\omega}{b_{11}^*} - \frac{b_{13}^*}{b_{11}^*} \frac{d_2\,\omega}{d_1^2 + d_2^2\omega^2} \right] \sin\frac{2n+1}{2l}\pi x;$$

$$\varphi_{w1} = -\mathrm{arctg}\left(F_p/E_p\right),\ \ \varphi_{u1} = \mathrm{arctg}\left(B_p/A_p\right).$$

The functions introduced into consideration $\Pi_{w1}(\omega, x)$ and $\Pi_{u1}(\omega, x)$ are the frequency-dependent distribution functions of the amplitudes of the deflection and the longitudinal movement of the first bearing layer along the channel. Note that using (17), we can write similar functions for deflection and longitudinal movement of the second bearing layer of plate.

Conclusion. A mathematical model is proposed for modeling bending and longitudinal vibrations of the channel wall as a three-layered plate possessing compressible core and interacting with a pulsating layer of a viscous incompressible fluid. The expressions for the elastic displacements of the layers of the plate, which completely determine its intensely deformed state, are determined. Frequency-dependent deflection amplitude distribution functions are constructed Π_{w1} (ω, x) and longitudinal movement Π_{u1} (ω, x) the first bearing layer, allowing to analyze the dynamic response of the channel wall. In particular, these functions with a fixed value of the longitudinal coordinate represent the amplitude-frequency characteristics of the corresponding cross-section of the channel wall. It can be noted that the proposed functions allow us to investigate hydroelastic vibrations of the channel wall and determine the distribution of fluid pressure in the channel. For example, they can be used to determine its resonant vibration frequencies and the corresponding amplitudes of longitudinal displacements, deflections of the bearing layers of the plate and pressure in the liquid. In addition, the developed model can be used for the further development of methods for non-destructive monitoring of the state of three-layered walls of channels filled with a pulsating viscous fluid. In particular, if, with the well-known harmonic law of pressure pulsation at the ends of the channel, in some fixed section, the amplitude-frequency characteristics of the deflection and longitudinal displacement of its second bearing layer are experimentally determined, then using relation (17), we can recalculate the characteristics the first bearing layer of the channel. Comparing the result obtained with the previously known reference result, one can judge the state of the channel wall. Thus, the developed mathematical model can be used both for studying the dynamic response of three-layered channel walls, in various devices and assemblies and for developing non-destructive testing technologies for three-layered structural elements in contact with fluid layers in lubrication and damping systems or cooling according to the parameters of forced oscillations of their bearing layers.

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