UDC 519.876.5 DOI: 10.18698/0236-3933-2021-4-93-108

PROBLEM OF MATHEMATICAL MODEL ADEQUACY IN ASSESSING THE SEISMIC RISK

V.A. Minaev m1va@bmstu.ru
R.O. Stepanov stepanovr@bmstu.ru
A.O. Faddeev fao1@mail.ru

Bauman Moscow State Technical University, Moscow, Russian Federation

Abstract

The problem of mathematical models' adequacy in assessing seismic risk is considered. It is demonstrated that the currently used methods of testing such models make it possible to assess only the consistency of simulation and real data by counting the number of earthquake epicenters that appear in the areas with increased values of the fields with various indicators. The paper proposes a fundamentally new approach to testing adequacy of the seismic risk assessment models based on examining statistical hypotheses. Application of this approach is considered in a seismic risk assessment model for the territory of Armenia and the adjacent regions. Practical implementation of the proposed approach and the results obtained convincingly confirm that the tested mathematical model is adequate. Normality of the seismic risk values general set distribution calculated by the probabilistic model for Armenia and the adjacent territories is presented. Correlation coefficients of theoretical and empirical frequencies distribution are 0.75–0.99. It is shown that adequacy of the seismic risk assessment probabilistic model should be checked taking into account the earthquake abyssal levels. Conclusions are provided on operability and possibility of further use of the considered method in checking adequacy of assessing the seismic risk mathematical models

Keywords

Mathematical model, model adequacy, seismic risk, statistical hypothesis, statistical criterion

Received 13.01.2021
Accepted 16.03.2021

Introduction. The current stage in the development of scientific research devoted to the problems of analysis, assessment and forecasting the hazardous endogenous geological processes is connected to development and application of rather complex mathematical models that make it possible to evaluate seismic and geodynamic risks for territories of different scale and geological structure [1–7].

However, the problem of the developed mathematical models' adequacy for assessing seismic and geodynamic risks is still poorly elaborated. At the same time, its solution is extremely important, since proving their adequacy makes it possible to unambiguously resolve the issue of such models' acceptability and possibility of their introduction for analytical and prognostic purposes.

As of today, the problem of acceptability of one or another mathematical model for assessing seismic risk is solved as follows [4]. On the basis of calculations performed according to a mathematical model for assessing seismic (or geodynamic) risks for a certain area under study, a distributed field of this risk indicators is constructed in the form of isolines with a certain section (step). When assessing seismic risk, only earthquakes are taken into consideration. While assessing geodynamic risk, not only earthquakes are taken into account, but also other hazardous geodynamic phenomena, such as creep, landslides, sinkholes, subsidence and other dangerous phenomena and processes. As a rule, shear stresses, or relative density of the geological environment deformable rock potential energy (deterministic models), or probability of seismic (or hazardous geodynamic) event (probabilistic and fuzzy models) are used as the indicators [4]. For definiteness, let us only assess the seismic risks.

Then, spatial distribution of the earthquake epicenters that already occurred over a certain period of time, for example, over the past 50 years is applied to the study area projected onto the "daytime" surface of the Earth.

After that, the number of earthquake epicenters is calculated, without taking into account their magnitude, that fell on the areas of the study territory outlined by the distributed indicator field increased values [1, 4]. In case of the probabilistic field, probability values exceeding 0.65 are taken as the increased values [4]. Magnitude of the indicator field increased value having no probabilistic nature, for example, the shear stress field, is understood to be such a value, at which the assessed hazardous natural phenomenon or process is taking place.

And, finally, the ratio is found of the number of earthquake epicenters that appear in the areas with increased values of the indicator field to the total number of all earthquakes that occurred in the study territory during the period under consideration. In this case, it is not the adequacy of a model that is assessed, but its quantitative compliance with certain real facts (data). In our case it is distribution of the earthquake epicenters that already occurred, i.e., consistency of model and real data is assessed, in fact, under certain accepted conditional constraints.

Let us consider an example [4], in which, according to a deterministic mathematical model that takes into account the influence of anomalous gravity field in isostatic reduction and disturbance at the Moho boundary (Mohorovičić boundary), seismic risk is assessed for the central part of the East European

Platform. And equipotential distribution of shear stresses in the lithosphere is constructed in this territory (Fig. 1). Red circles indicate the earthquake epicenters that already occurred in this area over the past 100 years.

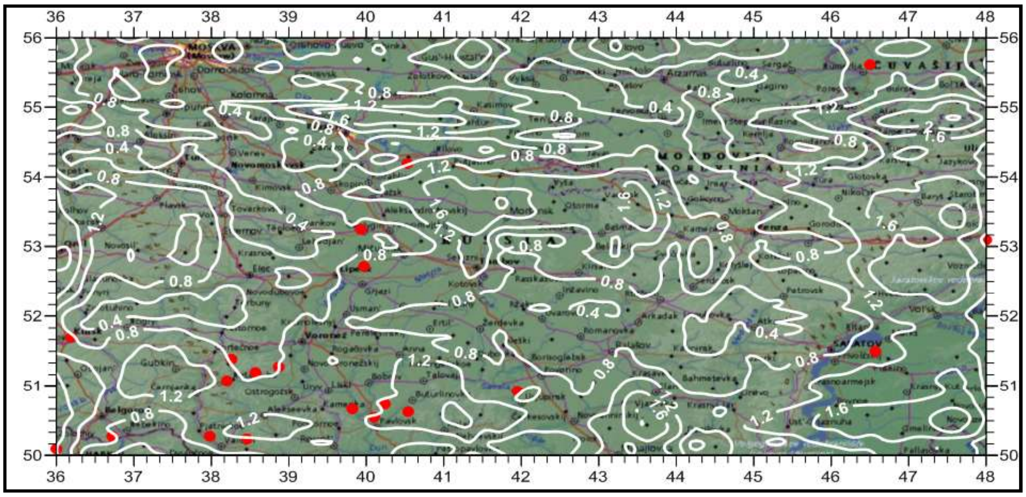

Fig. 1. Equipotential distribution of shear stresses in the lithosphere of the East European Platform central part calculated by the model, taking into account gravitational field anomalies in isostatic reduction and disturbances at the Moho boundary; cross-section of isolines is 0.4 MPa

On the basis of calculations performed according to the model, the range of variation in the shear stress values was determined, namely 0–2.4 MPa. Let us assume that the low-risk areas are those with shear stress values in the range of 0–0.8 MPa, medium-risk areas — in the range of 0.8–1.6 MPa, and high-risk areas — in the range of 1.6–2.4 MPa. Then, we obtain that the assessed territories are areas contoured by isolines with the shear stress increased values of over 0.8 MPa.

In this case, consistency of simulation and real data under the accepted conditional constraints is 15 : 20 = 0.75, since 15 out of 20 earthquake epicenters fell on the areas of shear stress increased values [4].

Nothing could be said about this model adequacy, since the considered approach is not providing any parameter that would allow its quantitative assessment. This paper proposes a method for testing the seismic risk assessment model adequacy based on evaluation of the statistical hypotheses [8, 9].

Statistical approaches to testing the adequacy of mathematical models in seismic risk assessment. Adequacy of the model, i.e., its compliance with simulated phenomenon or process, could really be tested using statistical criteria.

The procedure of adequacy assessment is based on comparing measurements obtained on a real system and results of experiments on a model, and it could be carried out in different ways. The most common of them are the following [8, 10, 11]: 1) by the average value of model and system responses; 2) according to the model response deviation variances from the system response average value; 3) by the maximum value of the model response relative deviations from the system responses.

These methods are quite close to each other; however, not all of them could be equally effectively applied in different systems.

Thus, the second and third methods are most successfully applied in relation to simulating the complex systems, the first method is the most effective for systems of medium complexity [8, 11, 12].

When seismic risks are investigated, geological environment of a certain extent and depth is the real system. Having a very complex organization, nevertheless, it is incomparable according to this characteristic with artificially created technical constructions, for which the second and third methods of testing the model adequacy are more suitable. Note that the first method is suitable for testing models in the presence of a solid statistical database [10, 11].

Thus, when studying a region possessing significant statistics (occurred earthquake epicenters distribution), it is quite competent to introduce the first method in testing adequacy of the seismic risk assessment mathematical model, namely, by examining average values of the model responses (calculated by the seismic risk values model) and of the real system (earthquake magnitudes).

The first way to test adequacy of the seismic risk assessment mathematical model is nothing more than testing the hypothesis that "the means of two samples refer to the same population". Such a test is carried out on the basis of the Student's criterion (or *t*-test) making it possible to determine the probability that mean values in two samples refer to one and the same population [10, 11].

It is necessary to find out whether the compared samples, i.e., the sample of the seismic risk values and the sample of earthquakes magnitudes occurred in a specific territory, belong to general populations distributed according to the normal law, since to apply the Student's *t*-test, it is necessary that initial data have normal distribution [9, 13, 14].

The territory of Armenia with the adjacent regions was selected as the test sample. Probabilistic mathematical model for assessing the seismic risk was introduced in the context of these territories [4] (Fig. 2).

Fig. 2 provides equipotential distribution of the probabilistic seismic risk and distribution of the earthquake epicenters occurred in the 1993–2014 period (epicenters are marked with red circles) [15, 16].

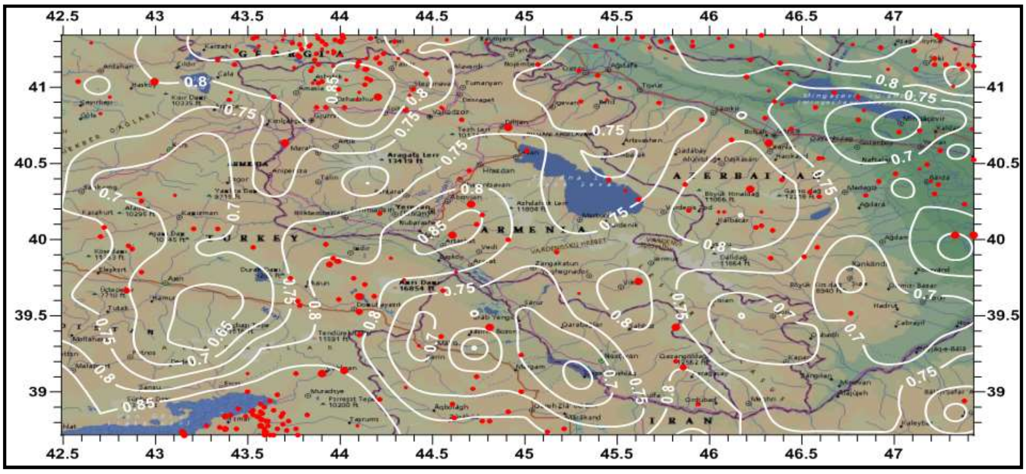

Fig. 2. Equipotential distribution of probabilistic seismic risk for Armenia and adjacent territories; contour section –0.05

All epicenters fall in the areas contoured by increased values of the probabilistic risk, i.e., in areas of 0.65 and more. That means that consistency between model and real data in this probabilistic case is 100 %, which makes us wonder, whether this approach is correct for regions exposed to orogenesis?

In fact, firstly, the study in terms of its geological structure belongs to the orogenic type seismically highly active regions. Within the indicated period of 22 years, 191 seismic events [15, 16] occurred in this territory, and only 20 seismic events occurred in the previously considered platform-type territory over the past 100 years [4].

Secondly, the described approach is not taking into account the fact that earthquakes differ significantly in magnitude and depth of their foci. Meanwhile, these factors significantly affect the seismic risk magnitude.

Analysis of the earthquake foci depth showed that it is distributed very unevenly (Fig. 3).

This leads to a conclusion that adequacy of the seismic risk assessment probabilistic model should be tested taking into consideration depth levels of the earthquake focus.

But let us return to testing the following hypothesis: "sample of the occurred earthquakes magnitude (see Fig. 2) belongs to the general population distributed according to the normal law". Such a test was carried out on the basis of using the Pearson's agreement criterion [9, 13, 14]. Results are provided in Table 1. In this case, general population case is understood as the entire aggregate of earthquakes occurred in the territory of Western Asia during the time interval under consideration.

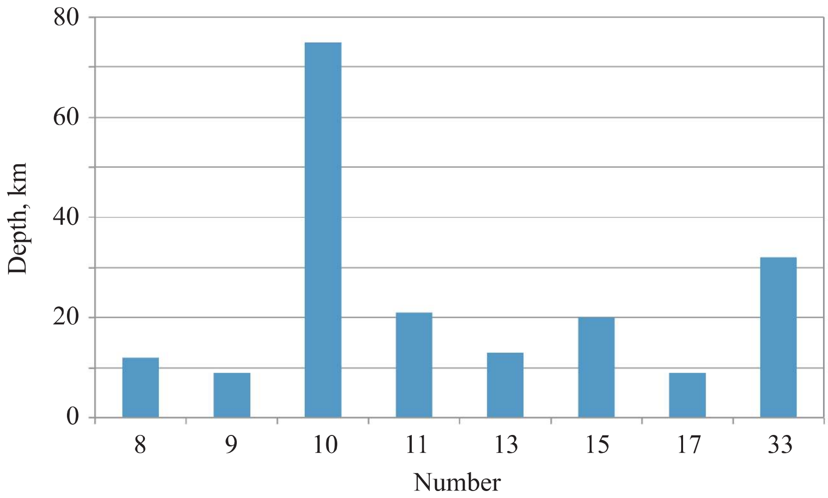

Fig. 3. Distribution of the earthquake foci occurrence depth in the 1993–2014 period on the territory of Armenia and the adjacent regions

Table 1

Results of testing the hypothesis of normal distribution of the occurred earthquakes magnitude

Magnitude (M_i)	$N_{emp\ i}$	u_i	$f(u_i)$	$N_{th\ i}$	$H_{cal\ i}$
3.5	3	−2.14415	0.04005	2.4360331	0.1305642
3.6	3	−1.82569	0.075357	4.5835711	0.547105614
3.7	9	−1.50724	0.128115	7.7925848	0.187081899
3.8	14	−1.18879	0.196805	11.970598	0.344048883
3.9	18	−0.87033	0.273166	16.615253	0.115407532
4.0	23	−0.55188	0.342589	20.837941	0.224326362
4.1	25	−0.23342	0.388221	23.613449	0.081416504
4.2	23	0.085032	0.397503	24.178024	0.057396761
4.3	20	0.403487	0.367755	22.368607	0.250811384
4.4	18	0.721941	0.307421	18.698805	0.02611551
4.5	14	1.040396	0.232201	14.123604	0.001081733
4.6	8	1.358851	0.158472	9.6390443	0.278706684
4.7	5	1.677305	0.097723	5.9440052	0.149923447
4.8	2	1.99576	0.05445	3.31193	0.51968498
4.9	2	2.314214	0.027413	1.6674003	0.066344328
5.0	3	2.632669	0.01247	0.7584996	6.62402943
5.1	1	2.951123	0.005126	0.3117652	1.519307288
				$\chi^2_{obs} = 11.12335254$	
				$\chi^2_{cr} = 23.68479131$	

First column of Table 1 shows magnitudes of the earthquakes, the second are the $N_{emp\ i}$ empirical (observed) frequencies; the third are the u_i normalized magnitudes; the fourth are the $f(u_i)$ function values; the fifth are the $N_{th\ i}$ theoretical frequencies, and the sixth are the $H_{cal\ i}$ calculated values obtained from the following relation [9, 14]:

$$H_{cal\ i} = \frac{(N_{emp\ i} - N_{th\ i})^2}{N_{th\ i}}. \qquad (1)$$

The u_i normalized magnitudes, the $f(u_i)$ function values and the $N_{th\ i}$ theoretical frequencies were calculated on the basis of the corresponding relations [9, 14]:

$$u_i = \frac{M_i - \bar{M}_s}{\sigma_s}, \qquad (2)$$

where $\bar{M}_s$ and σ_s are the sample mean and root-mean-square deviations calculated from the empirical sample of the M_i magnitudes and of the $N_{emp\ i}$ frequencies presented in Table 1;

$$\varphi(u_i) = \frac{1}{\sqrt{2\pi}} e^{-u_i^2/2}; \qquad (3)$$

$$N_{th\ i} = \frac{nh}{\sigma_s} \varphi(u_i), \qquad (4)$$

where n is the sample size (sum of all empirical frequencies); h is the step (difference between two adjacent options).

The lower right corner of Table 1 presents the χ^2_{obs} Pearson' agreement criterion observed value and its χ^2_{cr} critical value at the $\alpha = 0.05$ significance level, as well as the k = 14 number of degrees of freedom, where $k = s$ – 3 = 17 – 3 = = 14 (s is the number of observations in the sample):

$$\chi^2_{obs} = \sum_{i=1}^{17} \frac{(N_{emp\ i} - N_{th\ i})^2}{N_{th\ i}} \approx 11.12; \ \ \chi^2_{cr}(0.05; 14) \cong 23.68.$$

Comparing observed and critical values of the Pearson's agreement criterion, the $\chi^2_{obs} < \chi^2_{cr}$ is obtained. This inequality indicates that hypothesis of the earthquake general aggregate normal distribution in West Asia over the period of 1993–2014 is valid.

Fig. 4 presents graphs of theoretical and empirical frequencies distribution from Table 1. Correlation coefficient between theoretical and empirical frequencies is 0.98.

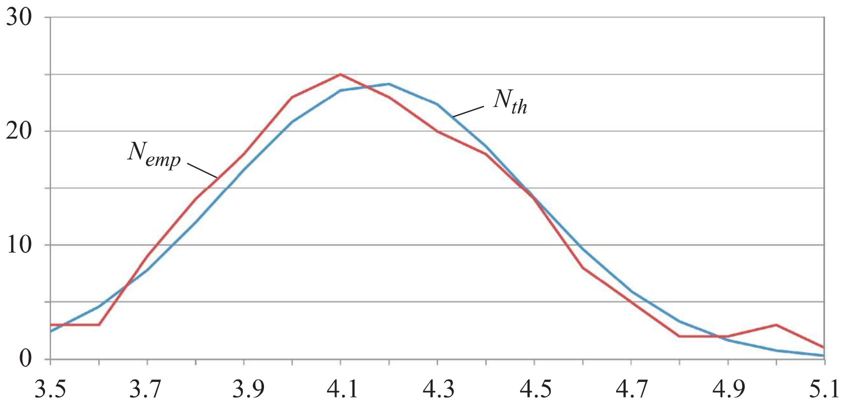

Fig. 4. Graphs of theoretical and empirical frequencies distribution by the earthquake magnitudes (see Fig. 2; see Table 1)

Let us check the hypothesis that “the seismic risk values sample calculated for each depth level according to the probabilistic model for the territory of Armenia and the adjacent regions belongs to general population distributed according to the normal law”. In this case, general population is understood as the entire set of probabilistic seismic risk values calculated for the geological environment volume corresponding to the territory of Armenia and the adjacent regions with a step of 1 km in depth and 0.01° in longitude and latitude. Let us first check this hypothesis for the depth level of 10 km, where the maximum number of earthquakes was observed (n = 75) (Fig. 5). Test results are presented in Table 2.

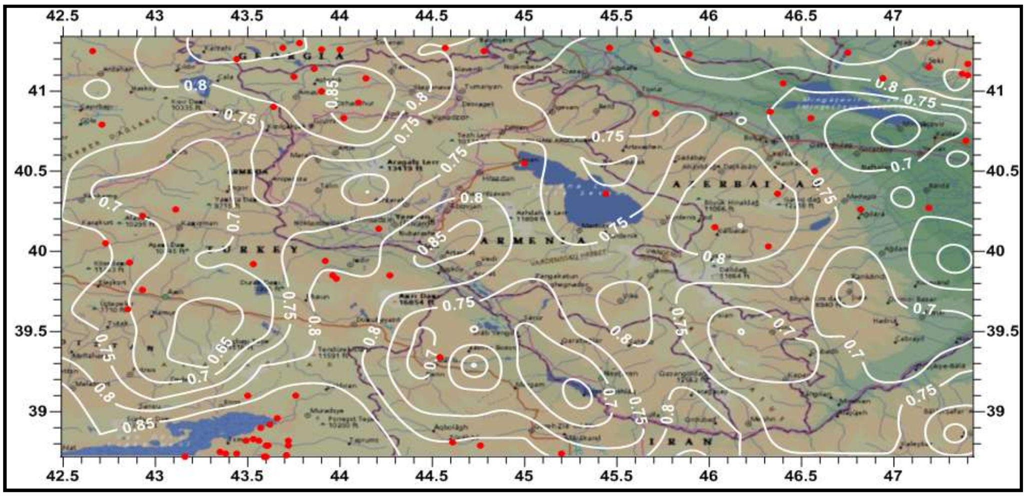

Fig. 5. Equipotential distribution of probabilistic seismic risk for Armenia and adjacent territories with earthquake epicenters at the depth of 10 km during the period of 1993–2014; contour section is 0.05

Table 2

Results of testing the hypothesis that the calculated seismic risks sample at the depth of 10 km belongs to normal distribution

x_i	x_{i+1}	x_i^*	N_{emp}	z_i	z_{i+1}	$\Phi(z_i)$	$\Phi(z_{i+1})$	P_i	N_{th}	H_{cal}
0.66	0.7	0.68	3	$-\infty$	−0.53	−0.5	−0.4463	0.0537	4.0275	0.26214
0.7	0.74	0.72	9	−0.53	−0.53	−0.446	−0.3554	0.0909	6.8175	0.69869
0.74	0.78	0.76	12	−0.53	−0.53	−0.355	−0.1915	0.1639	12.293	0.00696
0.78	0.82	0.8	17	−0.53	−0.52	−0.192	0.0239	0.2154	16.155	0.0442
0.82	0.86	0.84	16	−0.52	−0.52	0.0239	0.2291	0.2052	15.39	0.02418
0.86	0.92	0.89	8	−0.52	−0.51	0.2291	0.4265	0.1974	14.805	3.12786
0.92	0.96	0.94	10	−0.51	∞	0.4265	0.5	0.0735	5.5125	3.65309
									$\chi^2_{obs} = 7.81711$	
									$\chi^2_{cr} = 9.48773$	

The first and second columns of Table 2 show the left (x_i) and the right (x_{i+1}) boundaries of intervals, into which the entire range of seismic risks calculated using the probabilistic model is divided, the third column is the midpoints of the x_i^* intervals, and the fourth is the N_{emp} empirical frequencies.

The fifth and sixth columns present the z_i and z_{i+1} intervals' normalized boundaries calculated by formulas [9, 14]:

$$z_i = \frac{x_i - \overline{x}^*}{\sigma^*}; \quad z_{i+1} = \frac{x_{i+1} - \overline{x}^*}{\sigma^*}, \tag{5}$$

where $\overline{x}^*$ and σ^* are the sample mean and the sample root-mean-square deviation calculated for the x_i^* midpoints of intervals. When calculating the z_i and z_{i+1} values, the lowest value is assumed to be equal to $-\infty$, and the highest equal to $+\infty$.

The seventh and eighth columns of Table 2 show the Laplace function values from the z_i and z_{i+1} arguments, and the ninth is probabilities of the X value appearance in the intervals (x_i, x_{i+1}) calculated by the following formula:

$$P_i = \Phi(z_{i+1}) - \Phi(z_i). \tag{6}$$

The tenth column provides the N_{th} theoretical frequencies calculated according to the following formula:

$$N_{th\,i} = nP_i, \tag{7}$$

where n is the sample size (sum of all frequencies).

The lower right corner of Table 2 demonstrates the χ^2_{obs} observed value of the Pearson's agreement criterion and its χ^2_{cr} critical value at the $\alpha = 0.005$ level of significance and the $k = 4$ number of degrees of freedom, where $k = s - 3 = 7 - 3 = 4$ (s is the number of intervals in the sample):

$$\chi^2_{obs} = \sum_{i=1}^{7} \frac{(N_{emp\,i} - N_{th\,i})^2}{N_{th\,i}} \approx 7.82; \quad \chi^2_{cr}\,(0.05;\; 4) \approx 9.49.$$

Comparing observed and critical values of the Pearson's agreement criterion, the $\chi^2_{obs} < \chi^2_{cr}$ is found. This testifies to the normal distribution of the general set of seismic risk values calculated using the probabilistic model for Armenia and adjacent territories at the geological environment depth of 10 km.

Fig. 6 shows the graphs of theoretical and empirical frequencies distribution from Table 2. Correlation coefficient between them is 0.75.

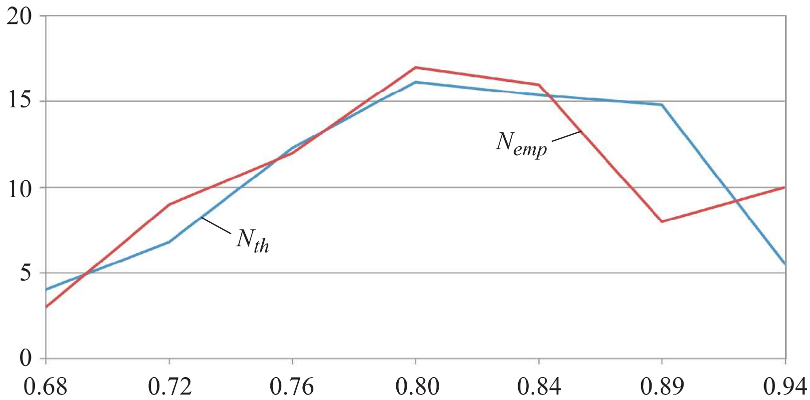

Fig. 6. Distribution of theoretical and empirical frequencies according to the probabilistic seismic risk values (see Table 2)

Note that correlation coefficient for the probability distribution densities calculated from theoretical and empirical frequencies from Table 2 is 0.99 (Fig. 7). Correlation coefficient for the probability distribution density functions, which values are calculated according to Table 1, is 0.94.

Thus, the sample of occurred earthquake magnitudes and the sample of seismic risk values corresponding to earthquakes at the depth of 10 km for the territory of Armenia and adjacent regions belongs to general populations distributed according to the normal laws.

Since samples of the seismic risk values for all other depth levels are taken from the same population, as the studied sample for the depth of 10 km, we assume that they could also be used to test the model adequacy at each depth level based on the Student's criterion.

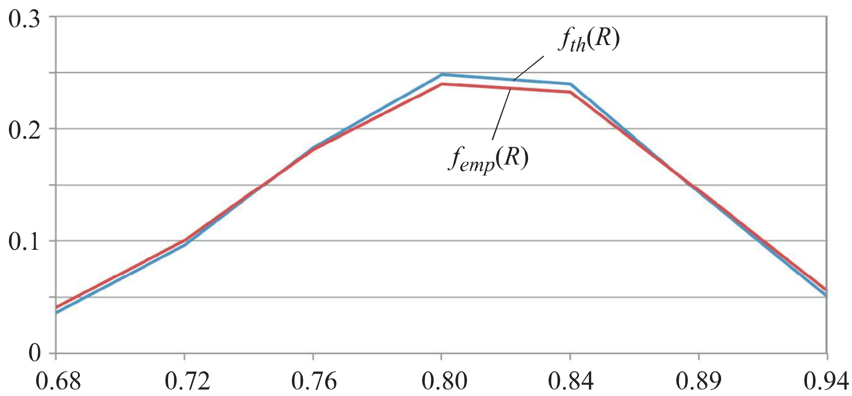

Fig. 7. Probability density distributions for the seismic risk values calculated according to theoretical and empirical frequencies (from Table 2)

Main stages of testing seismic risk assessment model adequacy. Let us consider the stages of testing adequacy of the seismic risk assessment mathematical model by average value of the model responses and of the geological environment using the Student's criterion (t-criterion).

Stage 1. Average value of the geological environment (mean magnitude value) is calculated as follows:

$$\bar{M} = \frac{1}{n_1} \sum_{i=1}^{n_1} M_i, \qquad (8)$$

where M_i is the i-th earthquake magnitude; n_1 is the initial data sample volume (number of earthquakes).

Stage 2. Mean value of the model responses (seismic risk mean value) is calculated as follows:

$$\bar{R} = \frac{1}{n_2} \sum_{i=1}^{n_2} R_i, \qquad (9)$$

where R_i is the i-th seismic risk value; n_2 is the model data sample volume (number of seismic risk values).

Stage 3. Sample and corrected dispersions are evaluated for initial and model data as follows:

$$D_M = \frac{1}{n_1} \sum_{i=1}^{n_1} (M_i - \bar{M})^2;$$
$$D_R = \frac{1}{n_2} \sum_{i=1}^{n_2} (R_i - \bar{R})^2, \qquad (10)$$

$$S_M^2 = D_M \frac{n_1}{n_1 - 1};$$
$$S_R^2 = D_R \frac{n_2}{n_2 - 1}. \quad (11)$$

If the corrected dispersions are different, it is necessary to first test the hypothesis of the general variances equality using the Fisher — Snedecor criterion [9, 13, 14] at the *a* certain significance level. If the hypothesis on the general dispersions equality is not rejected, we are able to proceed to the next stage.

Stage 4. The Student's *t*-criterion value is determined by the following formula [9, 13, 14]:

$$t_{obs} = \frac{\left|\bar{M} - \bar{R}\right|}{\sqrt{\frac{\left(n_1 S_M^2 + n_2 S_R^2\right)}{n_1 + n_2 - 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}. \quad (12)$$

Stage 5. The t_{obs} calculated value is compared with the tabular critical value of the t_{cr} Student's *t*-criterion considered at the *a* significance level with the $n_1 + n_2 - 2$ degrees of freedom. If $t_{obs} < t_{cr}$, the hypothesis is accepted, and the model is considered adequate at the *a* significance level.

Let us give a specific example of testing the mathematical model adequacy for assessing the probabilistic seismic risk at the depth of 9 km, where the sample has a small volume (9 seismic events).

Table 3 provides results of testing adequacy of the mathematical model for assessing the probabilistic seismic risk.

Table 3

Results of testing adequacy of the seismic risk assessment model for the territory of Armenia and adjacent regions at the 9 km depth level

Risks R_i	$(R_i - \bar{R})^2$	M_i	$(M_i - \bar{M})^2$
0.7271028	0.00522407	0.72549	0.00759442
0.7265632	0.00530236	0.803922	$7.59442 \cdot 10^{-5}$
0.8171399	0.0003154	0.784314	0.000802161
0.840081	0.00165653	0.843137	0.000930316
0.8470555	0.00227291	0.823529	0.000118663
0.8457315	0.00214841	0.843137	0.000930316
0.945699	0.02140911	0.921569	0.011866281
0.7890486	0.00010675	0.764706	0.002297312

End of the Table 3

Risks R_i	$(R_i - \bar{R})^2$	M_i	$(M_i - \bar{M})^2$
0.6560028	0.02055715	0.803922	$7.59442 \cdot 10^{-5}$
$\bar{R}$ = 0.7994	S_R^2 = 0.0074	$\bar{M}$ = 0.8126	S_M^2 = 0.0031
–	–	F_{obs} = 2.3892	t_{obs} = 1.0997
–	–	F_{cr} = 3.4381	t_{cr} = 2.1199

The first and the third columns of Table 3 show the seismic risk values calculated using the probabilistic model, as well as the reduced magnitudes of earthquakes occurred at the depth of 9 km within the 1993–2014 period. The given magnitudes are obtained by dividing the i-th earthquake magnitude by the maximum magnitude value registered at the study area during the testing time period, in our case it is 5.1.

The second and the fourth columns demonstrate the squared deviations of risk values and of reduced magnitudes from their mean values. Table 3 also presents mean values of the reduced $\bar{M}$ magnitudes and $\bar{R}$ risks, as well as values of the sample corrected variances of the reduced S_M^2 magnitudes and S_R^2 seismic risks.

Since the S_M^2 and S_R^2 sample corrected dispersions are different, hypothesis of the general dispersion equality was tested using the Fisher — Snedecor criterion at the $\alpha = 0.05$ significance level and the $k_1 = k_2 = 8$ degrees of freedom. Test results indicate that there is no reason to reject the null hypothesis on the equality of general dispersions, since $F_{obs} < F_{cr}$.

After that, the t_{obs} value is calculated using formula (12) and is compared with the t_{cr} value at the $\alpha = 0.05$ significance level and the degree of freedom equal to 16. Comparison of these values also does not provide grounds to reject the null hypothesis that the mean of two samples, i.e., values of seismic risks calculated using the mathematical model and the given magnitudes, refer to the same population.

All this makes it possible to conclude on adequacy of the mathematical model for assessing seismic risks at the territory of Armenia and the adjacent regions (see Fig. 2) for the depth level of 9 km.

Similar studies at the $\alpha = 0.05$ significance level were carried out for all other depth levels indicated in Fig. 2. Final results of practical assessments are presented in Table 4.

As follows from data in Table 4, the model is adequate for all depth levels, where seismic events were registered in the territory of Armenia and the adjacent regions within the 1993–2014 period.

Table 4

Results of testing adequacy of the seismic risk assessment model for the territory of Armenia and the adjacent regions

Depth, km	Degrees of freedom (k_1; k_2)	F_{obs}	F_{cr}	t_{obs}	t_{cr}	Model adequacy
8	(12; 12)	2.727868	2.81793	1.275611	2.073873	Adequate
9	(9; 9)	2.389204	3.438101	1.099744	2.119905	
10	(75; 75)	1.436776	1.469451	1.462171	1.976122	
11	(21; 21)	1.321116	2.124155	0.848185	2.021075	
13	(13; 13)	2.587551	2.686637	0.415018	2.063899	
15	(20; 20)	1.200479	2.168252	1.161736	2.024394	
17	(9; 9)	3.242837	3.438101	2.020844	2.119905	
33	(32; 32)	1.807658	1.822132	1.910198	1.998971	

Conclusions. The method used nowadays in testing mathematical models for assessing the seismic risks makes it possible only to demonstrate consistency of model and real data under certain conditional constraints and only for the platform-type territories. In addition, the applied method of testing the model and real data consistency is not taking into account magnitudes of the earthquakes occurred and depth levels, where seismic events took place.

A fundamentally new method for testing adequacy of the seismic risk assessment mathematical models based on statistical methods of testing statistical hypotheses is proposed, and its application in testing mathematical model for assessing the probabilistic seismic risk implemented in relation to the territory of Armenia and adjacent regions is considered in detail.

Practical implementation of the proposed method convincingly demonstrated that the tested probabilistic model appeared to be adequate for all depth levels, where the earthquakes took place, which, in turn, makes it possible to conclude on operability, acceptability and possibility of further using this method for testing adequacy of mathematical models in assessing the seismic risk.

Translated by K. Zykova

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Minaev V.A. — Dr. Sc. (Eng.), Professor, Chief Research Fellow, Arctic Program Directorate, Bauman Moscow State Technical University (2-ya Baumanskaya ul. 5, str. 1, Moscow, 105005 Russian Federation).

Stepanov R.O. — Cand. Sc. (Eng.), Assoc. Professor, Director of the Arctic Program Directorate, Bauman Moscow State Technical University (2-ya Baumanskaya ul. 5, str. 1, Moscow, 105005 Russian Federation).

Faddeev A.O. — Dr. Sc. (Eng.), Professor, Chief Research Fellow, Arctic Program Directorate, Bauman Moscow State Technical University (2-ya Baumanskaya ul. 5, str. 1, Moscow, 105005 Russian Federation).

Please cite this article in English as:

Minaev V.A., Stepanov R.O., Faddeev A.O. Problem of mathematical model adequacy in assessing the seismic risk. *Herald of the Bauman Moscow State Technical University, Series Instrument Engineering*, 2021, no. 4 (137), pp. 93–108.
DOI: https://doi.org/10.18698/0236-3933-2021-4-93-108